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2010 J. Phys.: Condens. Matter 22 025601

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Quantum Boltzmann equation study for the Kondo breakdown quantum critical point

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Received 25 September 2009, in final form 8 November 2009

Published 9 December 2009

Online at stacks.iop.org/JPhysCM/22/025601

Abstract

We develop the quantum Boltzmann equation approach for the Kondo breakdown quantum critical point, involved with two bands for conduction electrons and localized fermions. Particularly, the role of vertex corrections in transport is addressed, crucial for non-Fermi liquid transport with temperature linear dependence. Only one band of spinons may be considered for scattering with gauge fluctuations, and their associated vertex corrections are introduced in the usual way, where the divergence of self-energy corrections is cancelled by that of vertex corrections, giving rise to a physically meaningful result in the gauge invariant expression for conductivity. On the other hand, two bands should be taken into account for scattering with hybridization excitations, giving rise to coupled quantum Boltzmann equations. We find that vertex corrections associated with hybridization fluctuations turn out to be irrelevant due to the heavy mass of spinons in the so called decoupling limit, consistent with the diagrammatic approach showing non-Fermi liquid transport.

1. Introduction

Non-Fermi liquid transport phenomena near heavy fermion quantum critical points (QCPs) are one of the central interests in the field of strongly correlated electrons [1]. To reveal the scattering mechanism between charge carriers and critical fluctuations is the key to understanding the temperature linear resistivity [2], one of the hallmarks for non-Fermi liquid physics in quantum critical matter. Basically, the nature of critical modes, more precisely, the dynamical exponent z determining their dispersion relation $\omega \sim q^z$ and the scattering vertex between charge carriers and such critical modes are essential ingredients for the transport phenomena.

The standard model of quantum criticality in a metallic system is a $z = 2$ critical theory, often referred to as the Hertz–Moriya–Millis (HMM) theory [3]. Fortunately, many heavy fermion compounds have been shown not to follow the spin-density-wave (SDW) theoretical framework, where the temperature linear resistivity [4], divergent Grüneisen ratio with an exponent $2/3$ [5], Fermi surface reconstruction at the

QCP [6, 7], and the presence of localized magnetic moments at the transition towards magnetism [8] seem to support a more exotic scenario. An interesting suggestion is that the heavy fermion quantum transition is analogous to an orbital selective Mott transition [9–12], where only the f -electrons experience the metal–insulator transition, identified with a breakdown of the Kondo effect. This Kondo breakdown scenario differs from the HMM theory [3], in the respect that the whole heavy Fermi surface is destabilized at the QCP in the former case while only hot regions connected by SDW vectors become unstable in the latter.

The nature of the Kondo breakdown QCP turns out to be multi-scale [11, 12]. The dynamics of hybridization fluctuations is described by the $z = 3$ critical theory due to the Landau damping of electron-spinon polarization above an intrinsic energy scale E^* , while by a $z = 2$ dilute Bose gas model below E^* . The energy scale E^* originates from mismatch of the Fermi surfaces of conduction electrons and spinons, shown to vary from $\mathcal{O}(10^0)$ to $\mathcal{O}(10^2)$ mK. Based on the $z = 3$ quantum criticality, both the logarithmic divergent

specific heat coefficient and a power-law diverging thermal expansion coefficient was shown to fit successfully, giving rise to the divergent Grüneisen ratio with an exponent $2/3$ [13].

Recently, we showed that not only electrical resistivity but also thermal resistivity shows quasi-linear temperature dependence around the Kondo breakdown QCP due to scattering with $z = 3$ critical hybridization fluctuations [14], based on the Kubo formula where diagrammatic calculations were performed in the $1/N$ expansion with the spin degeneracy N . An important point is that vertex corrections for scattering with hybridization fluctuations can be neglected, a unique feature of the two band model, resulting from the heavy mass of spinons [11, 12]. This allows us to replace the transport time with the scattering time for such a process. Then, only the self-energy correction will be involved for transport, causing the non-Fermi liquid resistivity.

In this paper we clarify the issue related with vertex corrections for the transport phenomena at the Kondo breakdown QCP, based on quantum Boltzmann equations where vertex corrections are introduced naturally. An important feature is the emergence of coupled quantum Boltzmann equations for the distribution function of each band. In the heavy fermion phase described by condensation of Kondo bosons, only the lowest heavy fermion band may be taken into account, resulting in Fermi liquid transport owing to the absence of scattering with gapless fluctuations. In the fractionalized Fermi liquid phase of the Kondo breakdown scenario [10], hybridization fluctuations are gapped, leaving the two bands decoupled in the low energy limit and allowing us to consider the two quantum Boltzmann equations independently. On the other hand, at the Kondo breakdown QCP critical hybridization fluctuations force the two quantum Boltzmann equations to be coupled, requiring both distribution functions to be taken into account on equal footing. In this situation the quantum Boltzmann equation study has not been performed yet, at least for the heavy fermion QCP, as far as we know.

The present paper is organized as follows. In section 2 we introduce an effective field theory for the Kondo breakdown QCP. In section 3 we examine the electrical resistivity based on the coupled quantum Boltzmann equations, where the quantum Boltzmann equation study is reviewed for the $U(1)$ gauge theory of the one band model and its extension to the two band model is derived. In section 4 we summarize our results.

2. An effective field theory for the Kondo breakdown quantum critical point

We start from the Anderson lattice model in the large- U limit

$$L = \sum_i c_{i\sigma}^\dagger (\partial_\tau - \mu) c_{i\sigma} - t \sum_{\langle ij \rangle} (c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.}) + V \sum_i (d_{i\sigma}^\dagger c_{i\sigma} + \text{h.c.}) + \sum_i d_{i\sigma}^\dagger (\partial_\tau + \epsilon_f) d_{i\sigma} + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j, \quad (1)$$

where $c_{i\sigma}$ and $d_{i\sigma}$ are conduction electrons with a chemical potential μ and localized electrons with an energy level ϵ_f . The

last spin-exchange term is introduced for competition with the hybridization term of V .

Resorting to the $U(1)$ slave-boson representation $d_{i\sigma} = b_i^\dagger f_{i\sigma}$ with the single occupancy constraint $b_i^\dagger b_i + f_{i\sigma}^\dagger f_{i\sigma} = SN$ to take strong correlations with $S = 1/2$, one can rewrite equation (1) with

$$L = \sum_i c_{i\sigma}^\dagger (\partial_\tau - \mu) c_{i\sigma} - t \sum_{\langle ij \rangle} (c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.}) + V \sum_i (b_i f_{i\sigma}^\dagger c_{i\sigma} + \text{h.c.}) + \sum_i b_i^\dagger \partial_\tau b_i + \sum_i f_{i\sigma}^\dagger (\partial_\tau + \epsilon_f) f_{i\sigma} + \frac{J}{N} \sum_{\langle ij \rangle} (f_{i\sigma}^\dagger \chi_{ij} f_{j\sigma} + \text{h.c.}) + i \sum_i \lambda_i (b_i^\dagger b_i + f_{i\sigma}^\dagger f_{i\sigma} - 1) + \frac{J}{N} \sum_{\langle ij \rangle} |\chi_{ij}|^2, \quad (2)$$

where b_i and $f_{i\sigma}$ are holons and spinons, associated with hybridization and spin fluctuations, respectively. The spin-exchange term for the localized orbital is decomposed via the exchange hopping processes of spinons, where χ_{ij} is a hopping parameter for the decomposition. λ_i is a Lagrange multiplier field to impose the constraint, and N is the number of fermion flavours with $\sigma = 1, \dots, N$.

Performing the saddle-point approximation of $b_i \rightarrow b$, $\chi_{ij} \rightarrow \chi$, and $i\lambda_i \rightarrow \lambda$, one finds an orbital selective Mott transition as the breakdown of the Kondo effect at $J \approx T_K$, where a spin liquid Mott insulator ($\langle b_i \rangle = 0$) results in $J > T_K$ while a heavy fermion Fermi liquid ($\langle b_i \rangle \neq 0$) results in $T_K > J$ [10–12]. Here, $T_K = D \exp(-\frac{\epsilon_f}{N\rho_c V^2})$ is the single impurity Kondo temperature, where $\rho_c \approx (2D)^{-1}$ is the density of states for conduction electrons with the half bandwidth D .

Quantum fluctuations should be incorporated for the critical physics at the Kondo breakdown QCP, where two kinds of bosonic collective modes will scatter two kinds of fermions, that is, conduction electrons and spinons. Gauge fluctuations corresponding to phase fluctuations of the hopping parameter $\chi_{ij} = \chi e^{ia_{ij}}$ are introduced to express collective spin fluctuations [15]. Hybridization fluctuations are critical, playing an important role for the Kondo breakdown QCP. Such four field variables lead us to the following effective field theory in the continuum approximation,

$$\mathcal{L}_{ALM} = c_\sigma^* (\partial_\tau - \mu_c) c_\sigma + \frac{1}{2m_c} |\partial_i c_\sigma|^2 + f_\sigma^* (\partial_\tau - \mu_f - ia_\tau) f_\sigma + \frac{1}{2m_f} |\partial_i - ia_i f_\sigma|^2 + b^* (\partial_\tau - \mu_b - ia_\tau) b + \frac{1}{2m_b} |\partial_i - ia_i b|^2 + \frac{u_b}{2} |b|^4 + V(b^* c_\sigma^* f_\sigma + \text{h.c.}) + \frac{1}{4g^2} f_{\mu\nu} f_{\mu\nu} + SN(\mu_b + ia_\tau), \quad (3)$$

where g is an effective coupling constant between matter and gauge fields, and several quantities, such as fermion band masses and chemical potentials, are redefined as follows

$$\lambda \rightarrow -\mu_b, \quad (2m_c)^{-1} = t, \quad (2m_f)^{-1} = J\chi, \\ \mu_c = \mu + 2dt, \quad -\mu_f = \epsilon_f + \lambda - 2Jd\chi.$$

Fermion bare bands ϵ_k^c and ϵ_k^f for conduction electrons and spinons, respectively, are treated in the continuum approximation as $\epsilon_k^c \approx -2dt + t(k_x^2 + k_y^2 + k_z^2)$ and $\epsilon_k^f \approx -2Jd\chi + J\chi(k_x^2 + k_y^2 + k_z^2)$. The band dispersion for hybridization can arise from high energy fluctuations of conduction electrons and spinons. Actually, the band mass of holons is given by $m_b^{-1} \approx NV^2\rho_c/2$, where ρ_c is the density of states for conduction electrons [11, 12]. Local self-interactions denoted by u_b can be introduced via non-universal short-distance-scale physics. Maxwell dynamics for gauge fluctuations appears from high energy fluctuations of spinons and holons.

Based on the effective Lagrangian, recent studies [11, 12] developed an Eliashberg theory for the Kondo breakdown QCP, where the momentum dependence in fermion self-energies and vertex corrections are neglected, allowing us to evaluate one loop-level quantum corrections fully self-consistently. Actually, this approximation was shown to be ‘exact’ in the large N limit [16]. The Eliashberg theory for hybridization fluctuations results in the $z = 3$ Kondo breakdown QCP, discussed in the introduction.

3. Quantum Boltzmann equation study

We examine electrical transport at the Kondo breakdown QCP based on quantum Boltzmann equations, where we assume that both hybridization and gauge fluctuations are in equilibrium and consider only fermion contributions, consistent with the one loop result for the transport coefficient [14]. Since we have two kinds of fermion excitations, we find coupled quantum Boltzmann equations for distributions of conduction electrons and spinons. Solving such coupled quantum Boltzmann equations, we find that the diagrammatic result is recovered in the so called ‘decoupling’ limit of these equations, where vertex corrections for scattering with hybridization fluctuations can be ignored, but those for scattering with gauge fluctuations should be introduced in the spinon conductivity.

Before we perform the quantum Boltzmann equation study for the Kondo breakdown QCP with two bands, we review this approach in the $U(1)$ gauge theory with one band in order to understand the role of vertex corrections in the transport coefficient [17, 18] and demonstrate that our treatment successfully recovers the known result [19, 20].

3.1. Application to $U(1)$ gauge theory for a spin liquid state

We apply the quantum Boltzmann equation to the transport problem of $U(1)$ gauge theory,

$$S_{\text{eff}} = \int d\tau \int d^d r \left\{ \psi_\sigma^\dagger (\partial_\tau - ia_\tau - \mu_\psi) \psi_\sigma + \frac{1}{2m_\psi} |(\partial_i - ia_i) \psi_\sigma|^2 \right\} + \int \frac{dv}{2\pi} \sum_q D(q, v) \left(\delta_{ij} - \frac{q_i q_j}{q^2} \right) \times a_i(q, v) a_j(-q, -v), \quad (4)$$

where $D(q, v) = (-i\gamma_\psi \frac{v}{q} + \chi_\psi q^2)^{-1}$ is the gauge propagator with the diamagnetic susceptibility χ_ψ and Landau damping coefficient γ_ψ .

One can obtain this effective field theory from the Hubbard model in the frustrated lattice based on the $U(1)$ slave-rotor representation [21], where charge fluctuations are gapped at half filling, but magnetic ordering is prohibited owing to the geometrical frustration, corresponding to a spin liquid Mott insulator. One also finds this effective field theory in the so called algebraic charge liquid for the anomalous normal state of high T_c cuprates, derived from the $U(1)$ slave-fermion representation, where spin fluctuations described by Schwinger bosons are gapped, but charged excitations represented by fermionic holons are gapless, allowing an anomalous metallic state due to scattering with gauge fluctuations [22, 23].

Compared to the effective field theory (equation (3)) for the Kondo breakdown QCP of the Anderson lattice model, this $U(1)$ gauge theory (equation (4)) is a simplified version since it does not have both holons and conduction electrons. In this section we focus on the mathematical structure, in particular, the gauge invariant expression for conductivity [19, 20] instead of the physical aspect, in order to prepare for the Boltzmann equation study of the Anderson lattice model.

We start from the quantum Boltzmann equation [24]

$$[\partial_\omega f(\omega)] \Gamma(k, \omega) [A(k, \omega)]^2 \mathbf{v}_k \cdot \mathbf{E} = I_{\text{coll}}(k, \omega), \quad (5)$$

where $\Gamma(k, \omega)$ and $A(k, \omega)$ are the imaginary parts of the retarded self-energy and retarded Green’s function, respectively, $f(\omega)$ is the Fermi–Dirac distribution function in equilibrium, \mathbf{v}_k is the velocity of fermions, and \mathbf{E} is an external electric field. $I_{\text{coll}}(k, \omega)$ is the collision term given by

$$I_{\text{coll}}(k, \omega) = \Sigma^>(k, \omega) G^<(k, \omega) - \Sigma^<(k, \omega) G^>(k, \omega), \quad (6)$$

where $\Sigma^{<,>}(k, \omega)$ and $G^{<,>}(k, \omega)$ are lesser and greater self-energies and Green’s functions, respectively. Using the identity of

$$\begin{aligned} & \Sigma^>(k, \omega) G^<(k, \omega) - \Sigma^<(k, \omega) G^>(k, \omega) \\ &= 2i\Gamma(k, \omega) G^<(k, \omega) - i\Sigma^<(k, \omega) A(k, \omega), \end{aligned}$$

where the lesser self-energy is given by

$$\begin{aligned} \Sigma^<(k, \omega) &= \sum_q \int_0^\infty \frac{dv}{\pi} \left| \frac{k \times \hat{q}}{m_\psi} \right|^2 \Im D(q, v) \\ &\times [\{n(v) + 1\} G^<(k + q, \omega + v) \\ &+ n(v) G^<(k + q, \omega - v)], \end{aligned} \quad (7)$$

with the Bose–Einstein distribution function $n(v)$ and the spectral function $\Im D(q, v)$ for gauge fluctuations in the one loop approximation, the lesser Green’s function is the only unknown function, determined by the quantum Boltzmann equation. In this problem it is defined as the Fourier transformation for relative coordinates, given by

$$G^<(r, t; R, T) = i \left\langle \psi_\sigma^\dagger \left(R - \frac{r}{2}, T - \frac{t}{2} \right) \times \psi_\sigma \left(R + \frac{r}{2}, T + \frac{t}{2} \right) \right\rangle$$

with the centre of mass coordinates R, T and relative ones r, t , where dynamics of the ψ_σ field is governed by equation (4). In the spatially homogeneous case and steady state, one can neglect the R, T dependence for the distribution function. This quantum Boltzmann equation is derived well in [24], based on the Schwinger–Keldysh formulation.

In the linear response regime we can expand the lesser Green’s function up to the first order in the electric field

$$G^<(k, \omega) = iA(k, \omega) \left[f(\omega) - \left(\frac{\partial f(\omega)}{\partial \omega} \right) \mathbf{E} \cdot \mathbf{v}_k \Lambda(k, \omega) \right], \quad (8)$$

where $\Lambda(k, \omega)$ is the distribution function out of, but near equilibrium due to the electric field. Inserting this ansatz into the lesser self-energy, we obtain the following expression for the lesser self-energy

$$\begin{aligned} \Sigma^<(k, \omega) = & i \sum_q \int_0^\infty \frac{dv}{\pi} \left| \frac{k \times \hat{q}}{m_\psi} \right|^2 \Im D(q, v) f(\omega) \\ & \times \{ [n(v) + f(\omega + v)] A(k + q, \omega + v) \\ & - [n(-v) + f(\omega - v)] A(k + q, \omega - v) \} \\ & + i \sum_q \int_0^\infty \frac{dv}{\pi} \left| \frac{k \times \hat{q}}{m_\psi} \right|^2 \Im D(q, v) \mathbf{E} \cdot \mathbf{v}_{k+q} \left(-\frac{\partial f(\omega)}{\partial \omega} \right) \\ & \times \left\{ [n(v) + f(\omega + v)] \frac{1 - f(\omega + v)}{1 - f(\omega)} \right. \\ & \times A(k + q, \omega + v) \Lambda(k + q, \omega + v) \\ & - [n(-v) + f(\omega - v)] \frac{1 - f(\omega - v)}{1 - f(\omega)} \\ & \left. \times A(k + q, \omega - v) \Lambda(k + q, \omega - v) \right\}, \quad (9) \end{aligned}$$

where we used the identities for thermal factors of fermions and bosons,

$$\begin{aligned} \{n(v) + 1\} f(\omega + v) &= f(\omega) \{n(v) + f(\omega + v)\}, \\ n(v) f(\omega - v) &= -f(\omega) \{n(-v) + f(\omega - v)\}, \end{aligned}$$

and

$$\begin{aligned} \{n(v) + 1\} \left(-\frac{\partial f(\omega + v)}{\partial \omega} \right) &= \{n(v) + f(\omega + v)\} \\ & \times \frac{1 - f(\omega + v)}{1 - f(\omega)} \left(-\frac{\partial f(\omega)}{\partial \omega} \right), \\ n(v) \left(-\frac{\partial f(\omega - v)}{\partial \omega} \right) &= -\{n(-v) + f(\omega - v)\} \\ & \times \frac{1 - f(\omega - v)}{1 - f(\omega)} \left(-\frac{\partial f(\omega)}{\partial \omega} \right). \end{aligned}$$

Inserting both the lesser Green’s function and self-energy into the quantum Boltzmann equation, we find

$$\begin{aligned} \Lambda(k_F, \omega) \approx & \frac{1}{2} A(k_F, \omega) + \frac{1}{2\Gamma(k_F, \omega)} \\ & \times \sum_q \int_0^\infty \frac{dv}{\pi} \left| \frac{k_F \times \hat{q}}{m_\psi} \right|^2 \Im D(q, v) \\ & \times \{ [n(v) + f(\omega + v)] A(k_F + q, \omega + v) \\ & - [n(-v) + f(\omega - v)] A(k_F + q, \omega - v) \} \\ & \times \left(\frac{\mathbf{v}_{k_F} \cdot \mathbf{v}_{k_F+q}}{v_F^2} \right) \Lambda(k_F, \omega), \quad (10) \end{aligned}$$

where the momentum is replaced with the Fermi momentum, k_F because usual transport phenomena occur near the Fermi surface except some topological quantities such as Hall conductivity [25] and frequency dependence in both the ‘vertex-distribution’ function $\Lambda(k_F, \omega)$ and thermal Fermi factor is simplified. In this expression the imaginary part of the self-energy or scattering rate is defined as

$$\begin{aligned} 2\Gamma(k, \omega) = & \sum_q \int_0^\infty \frac{dv}{\pi} \left| \frac{k \times \hat{q}}{m_\psi} \right|^2 \Im D(q, v) \\ & \times \{ [n(v) + f(\omega + v)] A(k + q, \omega + v) \\ & - [n(-v) + f(\omega - v)] A(k + q, \omega - v) \}. \quad (11) \end{aligned}$$

This approximation will be justified by the fact that it gives rise to the known result in the gauge theory context.

Introducing the relative angle θ between the initial k_F and final $k_F + q$ momenta, we obtain

$$\Lambda(k_F, \omega) = \frac{2\Gamma(k_F, \omega)}{2\Gamma_{1-\cos}(k_F, \omega)} A(k_F, \omega), \quad (12)$$

where

$$\begin{aligned} 2\Gamma_{1-\cos}(k_F, \omega) = & \frac{3}{2\Lambda^3} \int_0^\Lambda dq q^2 \int_{-1}^1 d \cos \theta [v_F^2 \cos^2(\theta/2)] \\ & \times \int_0^\infty \frac{dv}{\pi} \Im D(q, v) [1 - \cos \theta] \\ & \times \{ [n(v) + f(\omega + v)] A(k_F + q, \omega + v) \\ & - [n(-v) + f(\omega - v)] A(k_F + q, \omega - v) \}. \quad (13) \end{aligned}$$

In this expression \sum_q is replaced with $\frac{3}{2\Lambda^3} \int_0^\Lambda dq q^2 \int_{-1}^1 d \cos \theta$ in $d = 3$, where Λ is a momentum cutoff. The $1 - \cos \theta$ factor in $\Gamma_{1-\cos}(k_F, \omega)$ identifies $[2\Gamma_{1-\cos}(k_F, \omega)]^{-1}$ with the transport time $\tau_{tr}(\omega)$, dominantly capturing large angle scattering.

The electrical (charge) or number conductivity is expressed by the lesser Green’s function,

$$J_\mu^\psi = -i \int \frac{d^3k}{(2\pi)^3} \frac{k_\mu}{m_\psi} \int \frac{d\omega}{2\pi} G^<(k, \omega). \quad (14)$$

Inserting the near-equilibrium ansatz for the lesser Green’s function into this expression, we obtain the electrical conductivity

$$\sigma_{\mu\nu}(T) = \int \frac{d^3k}{(2\pi)^3} \int \frac{d\omega}{2\pi} v_{k\mu} v_{k\nu} \left(-\frac{\partial f(\omega)}{\partial \omega} \right) A(k, \omega) \Lambda(k, \omega) \quad (15)$$

where the equilibrium contribution does not generate currents and thus vanishes.

Inserting the vertex-distribution function into the conductivity expression and performing the momentum and energy integration, we reach the final form of the conductivity

$$\sigma(T) \approx \mathcal{C} N_F v_F^2 \tau_{tr}(T) \quad (16)$$

with $\mathcal{C} = \frac{N}{\pi} \int_{-\infty}^\infty dy \frac{1}{(y^2+1)^2}$, where N_F is the density of states at the Fermi surface and the transport time is $\tau_{tr}(T) = [2\Gamma_{1-\cos}(T)]^{-1}$, as emphasized before.

The transport time turns out to be $\tau_{tr}(T) \propto T^{-5/3}$, giving rise to $\sigma(T) \propto T^{-5/3}$ in $d = 3$, completely consistent with the previous study [19, 20]. An important point is that

although the self-energy correction due to gauge fluctuations is diverging at finite temperatures, the gauge invariant expression for the conductivity allows only the finite result, cancelling the divergence via the vertex correction [18]. $1 - \cos \theta$ guarantees such a cancellation. This is the power of the quantum Boltzmann equation, imposing the vertex correction naturally.

In this derivation gauge fluctuations are assumed to be in equilibrium. Generally speaking, their non-equilibrium distribution due to external fields should be introduced. Actually, phonon drag effects are well known in the electron-phonon system [24]. Recently, this issue was considered in the spin liquid context with $z = 3$ gauge fluctuations [20], where coupled quantum Boltzmann equations for spinon and photon distribution functions are derived. It was argued that such coupled transport equations can be decoupled in some cases, where such drag effects are subdominant, compared with fermion contributions.

The present formulation differs from the previous approach in the fact that we did not decompose the gauge field as in the study of [19, 20], where the low energy gauge field giving rise to divergence is neglected and only high energy gauge fluctuations are taken. Although the vertex-distribution function itself is not well defined, because the part corresponding to the scattering rate is divergent at finite temperatures, we found that such a decomposition is not necessary because the formal divergence should be cancelled in the last gauge invariant physical expression. This spirit goes exactly through that of the diagrammatic study.

3.2. Application to the Kondo breakdown QCP of the Anderson lattice model

In the Kondo breakdown scenario we have four kinds of field variables, corresponding to conduction electrons, spinons, holons (hybridization fluctuations), and gauge bosons (collective spin fluctuations). Our main assumption for the transport study based on the quantum Boltzmann equation approach is that both hybridization and gauge fluctuations are in equilibrium, as pointed out earlier. This assumption is justified by the diagrammatic study [14], where contributions from boson excitations are much smaller than fermion contributions, and by the Boltzmann equation study of the $U(1)$ gauge theory discussed in the previous section. As a result, we are allowed to have two coupled quantum Boltzmann equations,

$$[A_c(k, \omega)]^2 \partial_\omega f(\omega) \mathbf{E} \cdot \mathbf{v}_k^c \Gamma_c(k, \omega) = I_{\text{coll}}^c(k, \omega),$$

$$I_{\text{coll}}^c(k, \omega) = 2i\Gamma_c(k, \omega)G_c^<(k, \omega) - i\Sigma_c^<(k, \omega)A_c(k, \omega) \quad (17)$$

for conduction electrons and

$$[A_f(k, \omega)]^2 \partial_\omega f(\omega) \mathbf{E} \cdot \mathbf{v}_k^f \Gamma_f(k, \omega) = I_{\text{coll}}^f(k, \omega),$$

$$I_{\text{coll}}^f(k, \omega) = 2i\Gamma_f(k, \omega)G_f^<(k, \omega) - i\Sigma_f^<(k, \omega)A_f(k, \omega) \quad (18)$$

for spinons. The lesser Green's function for conduction electrons is given by

$$G_c^<(r, t; R, T) = i \left\langle c_\sigma^\dagger \left(R - \frac{r}{2}, T - \frac{t}{2} \right) c_\sigma \left(R + \frac{r}{2}, T + \frac{t}{2} \right) \right\rangle,$$

and that for spinons is defined as

$$G_f^<(r, t; R, T) = i \left\langle f_\sigma^\dagger \left(R - \frac{r}{2}, T - \frac{t}{2} \right) f_\sigma \left(R + \frac{r}{2}, T + \frac{t}{2} \right) \right\rangle,$$

where the dynamics of c_σ and f_σ fields are governed by the Kondo breakdown model, equation (3). The spatially homogeneous case and steady state are taken into account, allowing these distribution functions to be independent of the centre of mass coordinates R, T .

3.2.1. Contribution of conduction electrons. The lesser self-energy for conduction electrons arises from scattering with hybridization fluctuations, given by

$$\Sigma_c^<(k, \omega) = V^2 \sum_q \int_0^\infty \frac{d\nu}{\pi} \Im D_b(q, \nu) [n(\nu) + 1] \times G_f^<(k + q, \omega + \nu) + n(\nu)G_f^<(k + q, \omega - \nu) \quad (19)$$

in the Eliashberg framework. Since the spinon Green's function appears in the electron self-energy, the two quantum Boltzmann equations are coupled with each other. This coupling effect is the main character for the quantum Boltzmann equation of the Anderson lattice model at the QCP.

Inserting the lesser Green's function of spinons

$$G_f^<(k, \omega) = iA_f(k, \omega) \left[f(\omega) - \left(\frac{\partial f(\omega)}{\partial \omega} \right) \mathbf{E} \cdot \mathbf{v}_k^f \Lambda_f(k, \omega) \right] \quad (20)$$

into the electron lesser self-energy and the lesser Green's function for conduction electrons

$$G_c^<(k, \omega) = iA_c(k, \omega) \left[f(\omega) - \left(\frac{\partial f(\omega)}{\partial \omega} \right) \mathbf{E} \cdot \mathbf{v}_k^c \Lambda_c(k, \omega) \right] \quad (21)$$

into the quantum Boltzmann equation for conduction electrons, we obtain

$$\begin{aligned} \Lambda_c(k_F^c, \omega) &\approx \frac{1}{2}A_c(k_F^c, \omega) + \frac{V^2}{2\Gamma_c(k_F^c, \omega)} \\ &\times \sum_q \int_0^\infty \frac{d\nu}{\pi} \Im D_b(q, \nu) \left(\frac{\mathbf{v}_{k+q}^f \cdot \mathbf{v}_k^c}{v_k^c{}^2} \right) \\ &\times \{ [n(\nu) + f(\omega + \nu)] A_f(k_F^f + q, \omega + \nu) \\ &- [n(-\nu) + f(\omega - \nu)] A_f(k_F^f + q, \omega - \nu) \} \Lambda_f(k_F^f, \omega), \end{aligned} \quad (22)$$

where

$$\begin{aligned} 2\Gamma_c(k, \omega) &= V^2 \sum_q \int_0^\infty \frac{d\nu}{\pi} \Im D_b(q, \nu) \{ [n(\nu) + f(\omega + \nu)] \\ &\times A_f(k + q, \omega + \nu) - [n(-\nu) \\ &+ f(\omega - \nu)] A_f(k + q, \omega - \nu) \} \end{aligned} \quad (23)$$

is the scattering rate of conduction electrons and the same approximations as the case of the $U(1)$ gauge theory are utilized. It is important to notice that the vertex-distribution function for conduction electrons is related with that for spinons. We should know the vertex-distribution function for spinons.

3.2.2. *Contribution of spinons.* The lesser self-energy for spinon excitations results from scattering with both hybridization and gauge fluctuations, given by

$$\begin{aligned}\Sigma_f^<(k, \omega) &= \Sigma_f^{b<}(k, \omega) + \Sigma_f^{a<}(k, \omega), \\ \Sigma_f^{b<}(k, \omega) &= V^2 \sum_q \int_0^\infty \frac{d\nu}{\pi} \Im D_b(q, \nu) \{ [n(\nu) + 1] \\ &\quad \times G_c^<(k + q, \omega + \nu) + n(\nu) G_c^<(k + q, \omega - \nu) \}, \\ \Sigma_f^{a<}(k, \omega) &= \sum_q \int_0^\infty \frac{d\nu}{\pi} \left| \frac{k \times \hat{q}}{m_f} \right|^2 \Im D_a(q, \nu) \{ [n(\nu) + 1] \\ &\quad \times G_f^<(k + q, \omega + \nu) + n(\nu) G_f^<(k + q, \omega - \nu) \},\end{aligned}\quad (24)$$

where the lesser Green's function of conduction electrons appear in the hybridization-vertex-induced spinon self-energy while that of spinons arises in the self-energy correction via gauge fluctuations.

Inserting the lesser Green's functions for both conduction electrons and spinons into the lesser self-energy and quantum Boltzmann equation for spinons, we find

$$\begin{aligned}\mathbf{v}_F^f \Lambda_f(k_F^f, \omega) &\approx \frac{\mathbf{v}_F^f}{2} A_f(k_F^f, \omega) \\ &+ \frac{V^2}{2\Gamma_f(k_F^f, \omega)} \sum_q \int_0^\infty \frac{d\nu}{\pi} \Im D_b(q, \nu) \mathbf{v}_{k_F^f+q}^c \\ &\times \{ [n(\nu) + f(\omega + \nu)] A_c(k_F^c + q, \omega + \nu) \\ &- [n(-\nu) + f(\omega - \nu)] A_c(k_F^c + q, \omega - \nu) \} \Lambda_c(k_F^c, \omega) \\ &+ \frac{1}{2\Gamma_f(k_F^f, \omega)} \sum_q \int_0^\infty \frac{d\nu}{\pi} \left| \frac{k_F^f \times \hat{q}}{m_f} \right|^2 \Im D_a(q, \nu) \mathbf{v}_{k_F^f+q}^f \\ &\times \{ [n(\nu) + f(\omega + \nu)] A_f(k_F^f + q, \omega + \nu) \\ &- [n(-\nu) + f(\omega - \nu)] A_f(k_F^f + q, \omega - \nu) \} \Lambda_f(k_F^f, \omega),\end{aligned}\quad (25)$$

where

$$\begin{aligned}2\Gamma_f(k, \omega) &= V^2 \sum_q \int_0^\infty \frac{d\nu}{\pi} \Im D_b(q, \nu) \\ &\times \{ [n(\nu) + f(\omega + \nu)] A_c(k + q, \omega + \nu) \\ &- [n(-\nu) + f(\omega - \nu)] A_c(k + q, \omega - \nu) \} \\ &+ \sum_q \int_0^\infty \frac{d\nu}{\pi} \left| \frac{k \times \hat{q}}{m_f} \right|^2 \Im D_a(q, \nu) \{ [n(\nu) \\ &+ f(\omega + \nu)] A_f(k + q, \omega + \nu) \\ &- [n(-\nu) + f(\omega - \nu)] A_f(k + q, \omega - \nu) \} \\ &\equiv 2\Gamma_f^b(k, \omega) + 2\Gamma_f^a(k, \omega)\end{aligned}\quad (26)$$

is the scattering rate of spinons resulting from scattering with both hybridization $\Gamma_f^b(k, \omega)$ and gauge fluctuations $\Gamma_f^a(k, \omega)$. One can check equations (25) and (26), considering that the hybridization-induced part is basically the same as that of the quantum Boltzmann equation for conduction electrons and the gauge-fluctuation part coincides with that shown in the $U(1)$ gauge theory of section 3.2.1.

Inserting the vertex-distribution function (equation (22)) for conduction electrons into the vertex-distribution function (equation (25)) for spinons, we find the following expression

for spinons

$$\begin{aligned}\Lambda_f(k_F^f, \omega) &= \frac{1}{2} \left\{ A_f(k_F^f, \omega) + \frac{\Gamma_{f,\cos}^b(k_F^f, \omega)}{\Gamma_f(k_F^f, \omega)} A_c(k_F^c, \omega) \right\} \\ &+ \left\{ \frac{\Gamma_{f,\cos}^b(k_F^f, \omega)}{\Gamma_f(k_F^f, \omega)} \frac{\Gamma_{c,\cos}(k_F^c, \omega)}{\Gamma_c(k_F^c, \omega)} \right. \\ &\left. + \frac{\Gamma_{f,\cos}^a(k_F^f, \omega)}{\Gamma_f(k_F^f, \omega)} \right\} \Lambda_f(k_F^f, \omega)\end{aligned}\quad (27)$$

with

$$\begin{aligned}2\Gamma_{f,\cos}^b(k_F^f, \omega) &\equiv V^2 \frac{3}{2\Lambda^3} \int_0^\Lambda dq q^2 \\ &\times \int_{-1}^1 d \cos \theta_{cf} \left(\frac{v_F^c}{v_F^f} \cos \theta_{cf} \right) \\ &\times \int_0^\infty \frac{d\nu}{\pi} \Im D_b(q, \nu) \{ [n(\nu) \\ &+ f(\omega + \nu)] A_c(k_F^c + q, \omega + \nu) \\ &- [n(-\nu) + f(\omega - \nu)] A_c(k_F^c + q, \omega - \nu) \}, \\ 2\Gamma_{f,\cos}^a(k_F^f, \omega) &\equiv \frac{3}{2\Lambda^3} \int_0^\Lambda dq q^2 \int_{-1}^1 d \cos \theta_{ff} \cos \theta_{ff} \\ &\times \int_0^\infty \frac{d\nu}{\pi} [v_F^{f2} \cos^2(\theta_{ff}/2)] \Im D_a(q, \nu) \\ &\times \{ [n(\nu) + f(\omega + \nu)] A_f(k_F^f + q, \omega + \nu) \\ &- [n(-\nu) + f(\omega - \nu)] A_f(k_F^f + q, \omega - \nu) \}, \\ 2\Gamma_{c,\cos}(k_F^c, \omega) &\equiv V^2 \frac{3}{2\Lambda^3} \int_0^\Lambda dq q^2 \\ &\times \int_{-1}^1 d \cos \theta_{cf} \left(\frac{v_F^f}{v_F^c} \cos \theta_{cf} \right) \\ &\times \int_0^\infty \frac{d\nu}{\pi} \Im D_b(q, \nu) \{ [n(\nu) + f(\omega + \nu)] \\ &\times A_f(k_F^f + q, \omega + \nu) \\ &- [n(-\nu) + f(\omega - \nu)] A_f(k_F^f + q, \omega - \nu) \},\end{aligned}\quad (28)$$

where θ_{cf} represents an angle between the initial electron velocity v_F^c and final spinon velocity v_F^f , and θ_{ff} is defined in a similar way, but between spinons.

We obtain the spinon vertex-distribution function

$$\begin{aligned}\Lambda_f(k_F^f, \omega) &= \frac{1}{2} \{ \Gamma_f(k_F^f, \omega) A_f(k_F^f, \omega) \\ &+ \Gamma_{f,\cos}^b(k_F^f, \omega) A_c(k_F^c, \omega) \} \left\{ \Gamma_f^b(k_F^f, \omega) + \Gamma_{f,1-\cos}^a(k_F^f, \omega) \right. \\ &\left. - \Gamma_{f,\cos}^b(k_F^f, \omega) \frac{\Gamma_{c,\cos}(k_F^c, \omega)}{\Gamma_c(k_F^c, \omega)} \right\}^{-1},\end{aligned}\quad (29)$$

where

$$\begin{aligned}2\Gamma_{f,1-\cos}^a(k_F^f, \omega) &\equiv \frac{3}{2\Lambda^3} \int_0^\Lambda dq q^2 \int_{-1}^1 d \cos \theta_{ff} [1 - \cos \theta_{ff}] \\ &\times \int_0^\infty \frac{d\nu}{\pi} [v_F^{f2} \cos^2(\theta_{ff}/2)] \Im D_a(q, \nu) \{ [n(\nu) \\ &+ f(\omega + \nu)] A_f(k_F^f + q, \omega + \nu) \\ &- [n(-\nu) + f(\omega - \nu)] A_f(k_F^f + q, \omega - \nu) \}\end{aligned}\quad (30)$$

is identified with $[\tau_{ff}^a(\omega)]^{-1}$ as shown in the $U(1)$ gauge theory of section 3.2.1.

3.2.3. *Conductivity in the decoupling limit.* In the vertex-distribution function for spinons (equation (25)) we neglect the coupling term $\Lambda_c(k_F^c, \omega)$ as the zeroth order approximation for the transport study, named as the decoupling limit. One may understand the validity of this approximation, based on the fact that spinons are heavily massive, denoted by $\alpha \ll 1$, and scattering with conduction electrons will not affect their dynamics much. Then, we find

$$\Lambda_f(k_F^f, \omega) = \frac{1}{2} \frac{\Gamma_f(k_F^f, \omega) A_f(k_F^f, \omega)}{\Gamma_f^b(k_F^f, \omega) + \Gamma_{f,1-\cos}^a(k_F^f, \omega)}. \quad (31)$$

Inserting this expression into the spinon conductivity, we obtain

$$\sigma_f(T) = \frac{C N_F^f v_F^2}{2\Gamma_f^b(T) + 2\Gamma_{f,1-\cos}^a(T)}, \quad (32)$$

exactly the same as that of the diagrammatic study [14], where vertex corrections are introduced only for the scattering channel with gauge fluctuations. As discussed in the previous section, $z = 3$ gauge fluctuations give rise to divergence for self-energy corrections to spinons, but cancelled by vertex corrections, allowing the gauge invariant finite physical conductivity proportional to $\sim T^{-5/3}$ in $d = 3$. One may ask why the same situation does not happen for scattering with $z = 3$ hybridization fluctuations. Actually, such $z = 3$ dynamics of holons is cut by an intrinsic energy scale E^* , and scattering with $z = 2$ hybridization fluctuations below E^* does not cause the divergence for self-energy corrections.

The vertex-distribution function for conduction electrons becomes

$$\Lambda_c(k_F^c, \omega) \approx \frac{1}{2} A_c(k_F^c, \omega) + \frac{1}{2} \frac{\Gamma_{c,\cos}(k_F^c, \omega)}{\Gamma_c(k_F^c, \omega)} \times \frac{\Gamma_f(k_F^f, \omega) A_f(k_F^f, \omega)}{\Gamma_f^b(k_F^f, \omega) + \Gamma_{f,1-\cos}^a(k_F^f, \omega)}, \quad (33)$$

where scattering with spinons is incorporated through the vertex-distribution function for spinons because light conduction electrons can be much more affected. However, calling

$$\frac{\Gamma_{c,\cos}(k_F^c, \omega)}{\Gamma_c(k_F^c, \omega)} = \mathcal{O}(v_F^f/v_F^c) \approx \alpha \ll 1, \quad (34)$$

the second contribution in the electron vertex-distribution can be neglected. As a result, the conductivity from conduction electrons is free from vertex corrections, becoming

$$\sigma_c(T) = C \frac{N_F^c v_F^c{}^2}{2\Gamma_c(T)}, \quad (35)$$

which coincides with that of the diagrammatic study [14] showing $\Gamma_c(T) \sim T \ln(T/E^*)$ in the $z = 3$ critical regime.

The last task is to find an actual expression for the physical conductivity, referred as the Ioffe–Larkin composition rule [26]

$$\sigma(T) = \sigma_c(T) + \frac{\sigma_b(T)\sigma_f(T)}{\sigma_b(T) + \sigma_f(T)} \approx \sigma_c(T), \quad (36)$$

where $\sigma_b(T)$ is the holon conductivity, much smaller than fermion contributions, justifying the last approximation.

One may ask the role of the spinon conductivity for any physical response functions. Actually, it contributes to the physical thermal conductivity given by the corresponding Ioffe–Larkin composition rule

$$\frac{\kappa(T)}{T} \approx \frac{\kappa_c(T)}{T} + \frac{\kappa_f(T)}{T}, \quad (37)$$

where $\kappa_{c,f}(T)$ are thermal conductivity of conduction electrons and spinons, respectively, and holon contributions are also neglected. Assuming that the Wiedemann–Franz law holds for each fermion sector, proven to be correct at least in the one loop approximation [14], we find

$$\frac{\kappa_f(T)}{T} \approx \frac{\pi^2}{3} (\sigma_c(T) + \sigma_f(T)), \quad (38)$$

suggesting that the Wiedemann–Franz law should be violated due to the presence of additional entropy carriers, that is, spinons at the Kondo breakdown QCP in the low temperature limit, i.e.,

$$L(T) \equiv \frac{\kappa(T)}{T\sigma(T)} \approx L_0 \left(1 + \frac{\rho_f v_F^f}{\rho_c v_F^c} \right) \quad (39)$$

with $L_0 = \pi^2/3$, the value of the Fermi liquid. This result would be robust beyond our approximation because this expression includes just the density of states and velocity at the Fermi energy, thus expected to be governed by a conservation law.

4. Summary

In this paper we developed the quantum Boltzmann equation approach for the Kondo breakdown QCP, with which two bands for conduction electrons and localized fermions are involved, where scattering with $z = 3$ critical hybridization fluctuations and $z = 3$ gapless gauge bosons relaxes their dynamics. Our main problem was to understand the role of vertex corrections in their transport phenomena, crucial for the T -linear non-Fermi liquid resistivity in the $z = 3$ critical theory.

Only one band of spinons is involved for scattering with gauge fluctuations, and their associated vertex corrections are introduced in the usual way, demonstrated in the $U(1)$ gauge theory of section 3.1. Our treatment for gauge fluctuations is different from the previous study [19], in the respect that the vertex-distribution function is not well defined owing to the formal divergence associated with vertex corrections, but cancelled in the physical conductivity through self-energy corrections as it should be, consistent with the diagrammatic approach [18], while in the well defined previous study divergent contributions are thrown away in the vertex-distribution function. Of course, both approaches give the same result.

On the other hand, two bands should be taken into account for scattering with hybridization excitations, giving rise to coupled quantum Boltzmann equations. In the so called decoupling limit, where coupling effects are neglected for the Boltzmann equation of spinons while they are allowed for that

of conduction electrons, the vertex correction for conduction electrons associated with hybridization fluctuations turns out to be irrelevant due to the heavy mass of spinons, denoted by $\alpha \ll 1$. Results of the diagrammatic approach are recovered from the quantum Boltzmann equation approach in the decoupling limit.

The next task is what happens beyond the decoupling limit. Our preliminary analysis shows that vertex corrections seem to appear in the scattering channel with hybridization fluctuations. However, we have not found the corresponding diagram for such a correction at present. It remains as an interesting future study.

Acknowledgments

This work was supported by the French National Grant ANR36ECCEZZZ. K-S Kim was also supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (No. 2009-0074542).

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